Modeling LLM statements and dialogs

Dean Foster

Amazon & U Penn

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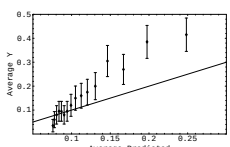
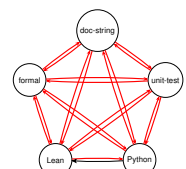
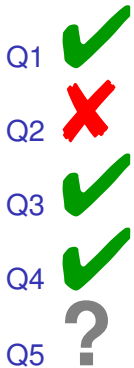




How often is an LLM correct?



Easy case

If all the questions are from GSM8K it is easy

Hard case

- Which is the best Lift, Waydoo or Fliteboard?
- My hand falls asleep when I'm biking, what is going on?
- Does M-convexity imply submodularity?
- . . .

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What does accuracy even mean?

Hard case

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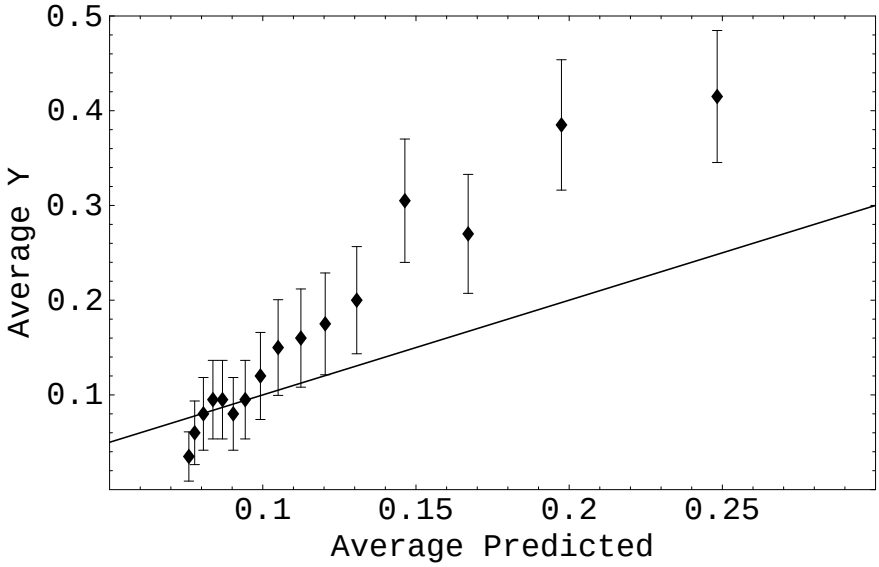
- My hand falls asleep when I'm biking, what is going on?
- Does M-convexity imply submodularity?
- . . .

What does accuracy even mean?
I'll argue "calibration" is the right interpretation.

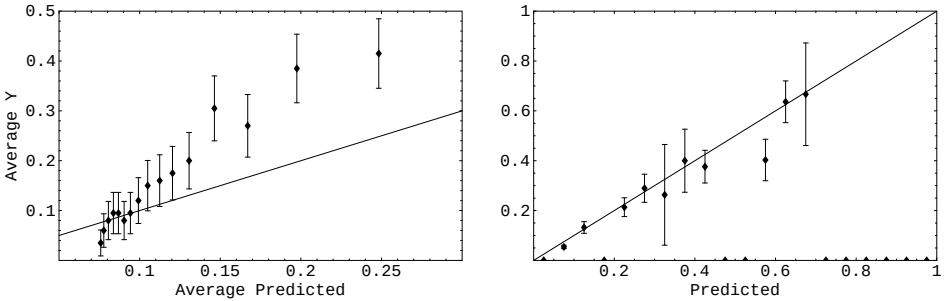
Calibration

- I been working on calibration since **1988**.
- Back then it was a niche game-theory question.
- Today it's the right way to answer "How often is an LLM correct?"

This is not calibrated



Anything easily fixed isn't calibrated



Fix the obvious problems!

Calibration is achievable

Theorem

A calibrated forecast exists.

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PROOF:

Apply the minimax theorem. QED

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Apply the minimax theorem. QED
(We'll give a different proof later, via online regression.)

Calibration limits LLM mis-alignment

Calibration is a low bar, so one all systems should achieve.

Reducing calibration to no-regret

- Many results in no-regret learning theory say we can make decisions in real time as well as we could in hindsight.
- We'll consider the least squares case.

Crazy calibration variable

Y	X_1	X_2	X_3	X_4
Y_1	X_{11}	X_{12}	X_{13}	X_{14}
Y_2	X_{21}	X_{22}	X_{23}	X_{24}
Y_3	X_{31}	X_{32}	X_{33}	X_{34}
Y_4	X_{41}	X_{42}	X_{43}	X_{44}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Y_t	X_{t1}	X_{t2}	X_{t3}	X_{t4}

Starting with our data that we observed up to time t

Crazy calibration variable

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$$\hat{\beta}_t = \arg\min_\beta \sum_{i=1}^{t} (Y_i - \beta' X_i)^2$$

We can fit $\hat{\beta}_t$ on everything up to time t

Crazy calibration variable

Y	X_1	X_2	X_3	X_4
Y_1	X_{11}	X_{12}	X_{13}	X_{14}
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Y_t	X_{t1}	X_{t2}	X_{t3}	X_{t4}
	$X_{t+1,1}$	$X_{t+1,2}$	$X_{t+1,3}$	$X_{t+1,4}$

$\hat{\beta}_t$

$$\hat{Y}_{t+1} = \hat{\beta}_t' X_{t+1}$$

From a new X_{t+1} we can compute $\hat{Y}_{t+1}$

Crazy calibration variable

Y	X_1	X_2	X_3	X_4	$\hat{\beta}$
Y_1	X_{11}	X_{12}	X_{13}	X_{14}	0
Y_2	X_{21}	X_{22}	X_{23}	X_{24}	$\hat{\beta}_1$
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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Y_t	X_{t1}	X_{t2}	X_{t3}	X_{t4}	$\hat{\beta}_{t-1}$

Looking at only the first part of the data, we can generate:

$$\hat{\beta}_0, \quad \hat{\beta}_1, \quad \hat{\beta}_2, \quad \hat{\beta}_3, \quad \hat{\beta}_4, \quad \ldots, \quad \hat{\beta}_{t-1}$$

Crazy calibration variable

Y	X_1	X_2	X_3	X_4	$\hat{\beta}$	$\hat{Y}$
Y_1	X_{11}	X_{12}	X_{13}	X_{14}	0	$\hat{Y}_1 = 0$
Y_2	X_{21}	X_{22}	X_{23}	X_{24}	$\hat{\beta}_1$	$\hat{Y}_2 = \hat{\beta}_1' X_2$
Y_3	X_{31}	X_{32}	X_{33}	X_{34}	$\hat{\beta}_2$	$\hat{Y}_3 = \hat{\beta}_2' X_3$
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Y_t	X_{t1}	X_{t2}	X_{t3}	X_{t4}	$\hat{\beta}_{t-1}$	$\hat{Y}_t = \hat{\beta}_{t-1}' X_t$

Each of these leads to a next round

$$\hat{Y}_1, \quad \hat{Y}_2, \quad \hat{Y}_3, \quad \hat{Y}_4, \quad \ldots, \quad \hat{Y}_t$$

Crazy calibration variable

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Theorem (Foster 1991, Forster 1999)

Such an on-line least squares forecast generates low regret:

$$\sum_{t=1}^{T} (Y_t - \hat{Y}_t)^2 - \min_{\beta} \sum_{t=1}^{T} (Y_t - \beta' X_t)^2 \leq O(\log(T))$$

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Works no matter what the X's are.

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Even if one of the X's were $\hat{Y}$!

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Theorem ($\Longrightarrow$ Foster and Kakade 2008, Foster and Hart 2018)

Adding the crazy calibration variable gives multi-calibration:

$$(\forall i) \quad \sum_{t=1}^{T} X_{t,i}(Y_t - \hat{Y}_t) = O(\sqrt{T \log(T)})$$

Progressive con (Sometimes called foot-in-the-door)

- $2 + 2 = 4$ ✔
- The sky is blue. ✔
- Another trivial statement. ✔
- . . . ✔
- . . . ✔
- . . . ✔ . . . ✔ . . . ✔

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- LLMs will not take your job.

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- . . . ✔ . . . ✔ . . . ✔
- LLMs will not take your job. ✘
  - Calibration says “Yes”
  - Multi-calibration says “no”

Multi-calibration is the protection we need

If we use multi-calibration, we are calibrated on:

- mathematics ✔
- history ✔
- factual questions ✔
- theorems ✔
- trivial questions ✔
- meaningful questions ✔

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(But not necessarily on historical$\times$factual$\times$trivial questions)

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- `bash`, `git`, file edits, the web, . . .
- different projects, different scales

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- no borrowing strength across surfaces.

Multi-calibration is exactly that guarantee — so we can let the agent loose and still trust each kind of action.

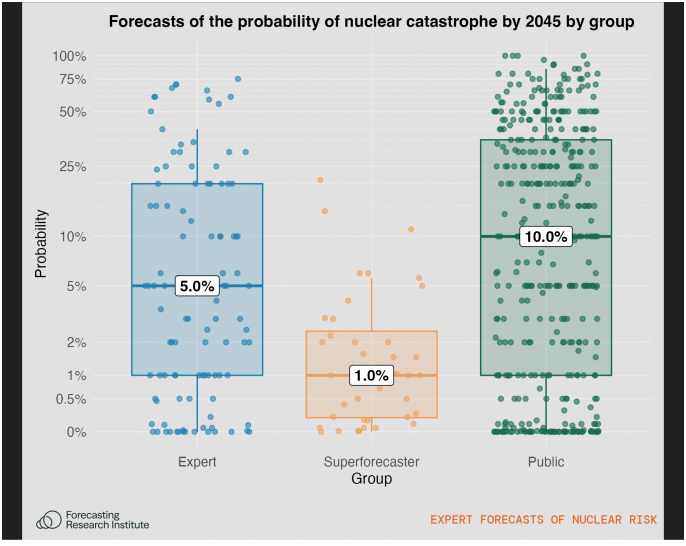
Feeding LLMs back and forth

- Feed one LLM's forecast into another.
- This generates a "dialog."
- Hart and I call this calibeating
- Each round the forecasts get better and better (Collina et al).
- Is there a limit?

Agreeing to disagree

- This back and forth converges:
  - Theorem by Bob Aumann (Nobel prize, “Agreeing to disagree.”)
  - Scott Aaronson did it for computers in 2001
  - LLMs proved it last week using Lean!
- Claim: We all agree at the end

We all agreement in the end: or maybe not



A typical dialog

- These aren't very interesting conversations:
  - He said: .9
  - She said: .7
  - He said: .7
  - She said: .6
  - ...
  - both say: .55
- Let's turn to more interesting dialogs

Calibration: how often it's right.

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Thinking: How to make it right.

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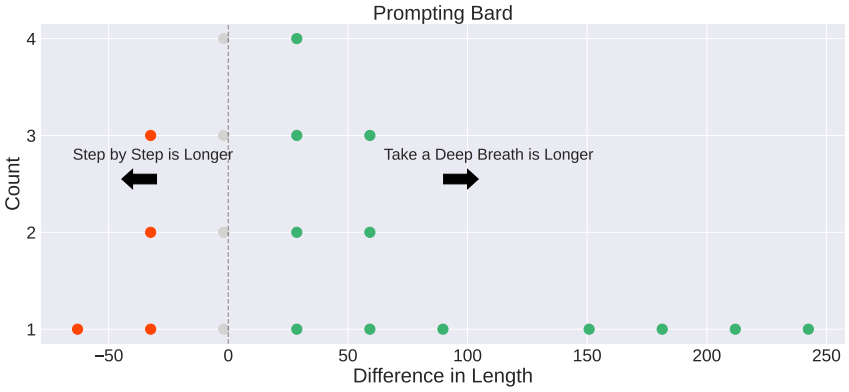
Thinking: How to make it right.

Checking: Is *this* answer right?

LLMs do better with more thinking



Contrasting native LLMs vs Chain of Thought

Theorem (Merrill and Sabharwal 2023)

(rephrased) An LLM can not answer questions in PSPACE. [arXiv]

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Other versions:

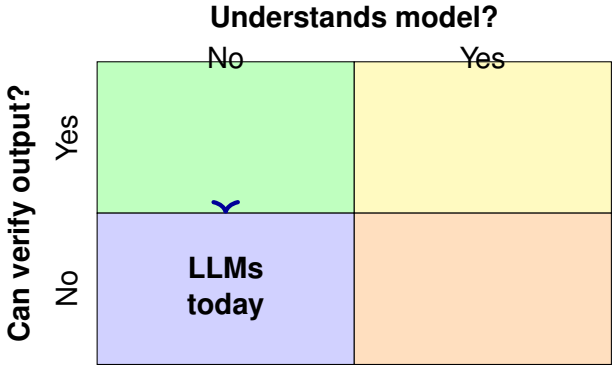
Theorem (Malach 2023)

A linear LLM can be trained to mimic a Turing machine using chain-of-thought. [arXiv]

Theorem (Giannou, Rajput, Sohn, Lee, Lee, and Papailiopoulos 2023)

Looped Transformers are general computers. [arXiv]

We can't open the box — but we can challenge it



Interactive proofs move LLMs *up*: verifiable without being transparent.

IP: a better kind of dialog

- “He said .9, she said .7” converges, but isn’t informative.
- Interactive proofs (Goldwasser–Micali–Rackoff): a bounded *verifier* interrogates a powerful *prover*.
- Model the human as a poly-time verifier, the LLM as a (near) exponential-time prover.

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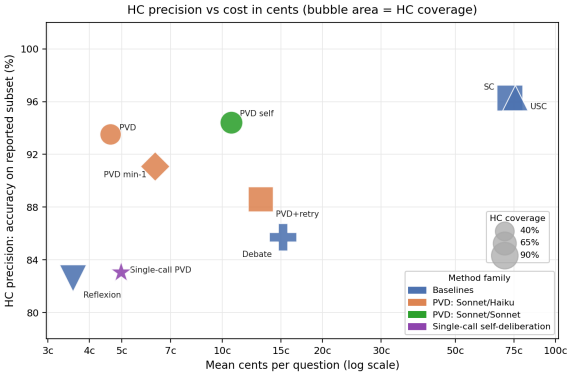
- Interactive proofs (Goldwasser–Micali–Rackoff): a bounded *verifier* interrogates a powerful *prover*.
- Model the human as a poly-time verifier, the LLM as a (near) exponential-time prover.
- The deliberation history *is* the proof: each round checks a partial claim.
- We don’t need a “faithful” explanation — we need *honesty*: claims stay consistent across rounds.
- Zero-knowledge: the model’s billions of parameters (the witness) are never revealed.

Prover–Verifier Deliberation

- Prover defends an answer through checkable sub-claims; verifier returns ACCEPT, CHALLENGE, or REJECT.
- "Accept + No Change" = the high-confidence subset: report it, abstain otherwise.
- ∼30pp precision gap between the confident and unsure answers.



A real IP dialog: “is this code leak-free?”

Claim: this repository has no memory leaks.

Verifier: check Package 1 — any leak source?

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Each round forces a *checkable artifact* — file, line, test, or counterexample. The process *is* the proof

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- On GPQA: ANC answers are 84–98% precise, $\sim$30pp above the rest — at $\sim$3 calls/question (vs. 8 for self-consistency).

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- Put the verifier *outside* its competence (Humanity's Last Exam, weak verifier): the gap *inverts* to $-$7pp.
  - It accepts exactly the questions it cannot evaluate.

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- Put the verifier *outside* its competence (Humanity's Last Exam, weak verifier): the gap *inverts* to -7pp.
  - It accepts exactly the questions it cannot evaluate.
- The collapse is itself the signal: we learn *when to stop trusting the protocol.* (Calibration, again.)

Alignment: IP and calibration

- P can check PSPACE
  - So if they outsmart us, we can still check them
- RP can be calibrated against anyone!
  - So no matter how far they outsmart us, we can still count and compute their calibration accuracy

What should LLM’s think about?

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Lean!

What you might have heard about *Lean* already

- Lean is used for formalizing mathematics:
  - Terry Tao is fascinated by Lean
  - He did a math project with 100s of people
  - They could all write Lean proofs
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What you might have heard about *Lean* already

- Lean is used for formalizing mathematics:
  - Terry Tao is fascinated by Lean
  - He did a math project with 100s of people
  - They could all write Lean proofs
  - The proofs were “checked” by simply being type checked in Lean
- The entire Cambridge undergraduate mathematics has been formalized in Lean
  - extensive library (mathlib) exists
- Formalization of latex to lean can be done
  - Formalization is a great LLM application

What does *Lean* math look like?

Here is "$\sqrt{2}$ is irrational" in Lean:

```
example {m n p : ℕ} (nnz : n ≠ 0) (prime_p : p.Prime) :
    m^2 ≠ p * n^2 := by
  intro sqr_eq
  have nsqr_nez : n^2 ≠ 0 := by simpa
  have eq1 : Nat.factorization (m^2) p
           = 2 * m.factorization p := by
  -- ...
```

What is *Lean*?

- Lean is a pure functional programming language — no side effects at all.
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- Lean is a pure functional programming language — no side effects at all.
  - This makes it easy to prove theorems about the code you write.
- It is also logic, based on type theory:
  - A theorem is a *type*; a proof is a *term* of that type.
  - (Lambda calculus and Turing machines were mathematics before they were code.)

What does *Lean* code look like? (here is Quick Sort)

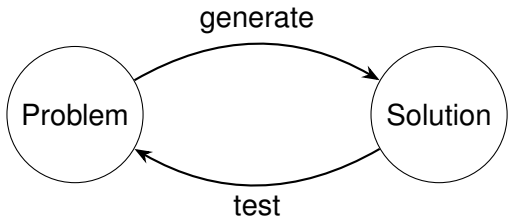
```
def qsort {α} (lt : α → α → Bool) : List α → List α
  | []     => []
  | h :: t =>
    let small := t.filter (fun x =>   lt x h)
    let large := t.filter (fun x => ! lt x h)
    qsort lt small ++ h :: qsort lt large
termination_by l => l.length
decreasing_by
  all_goals simp_wf
  all_goals
    apply Nat.lt_succ_of_le
    apply Nat.le_trans (List.length_filter_le _ _)
    simp
```

Writing Lean code is hard;

checking it is easy

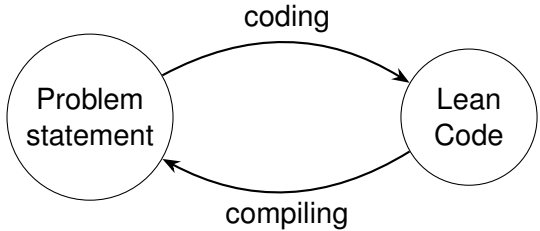
Generate and test: Mathematics

- CS:
  - Generation is easier than testing (P $\neq$ NP)
- mathematics:
  - Problem $\rightarrow$ solution: generation
  - Solution $\rightarrow$ problem: testing


generate
Problem
Solution
test

Generate and test

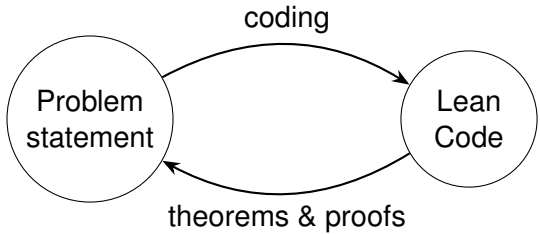
- Fits nicely into LLMs
  - Have one LLM generate a solution
  - Have a different one test the solution
  - Improve the generation
- We found disappointing results ([6])
  - Saturates after a few rounds
  - Effectively doubling the data size
  - Far from a polynomial – exponential split
  - Better models have a bigger gap, so the future might still be rosy for this approach


coding
Problem statement
Lean Code
compiling

Compiling only checks *termination*.

Compiling only checks *termination*.

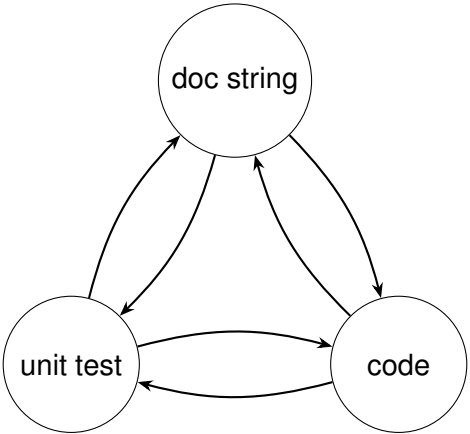
Type-checking $\neq$ correctness — that's the next arrow.


coding
Problem statement
Lean Code
theorems & proofs

Give LLMs problems they can more easily solve

- Have LLMs:
  - Write the goal of the program (aka doc string)
  - Write unit tests
  - Write code
- Have them check consistency between them
- No compiling necessary! All done by LLMs

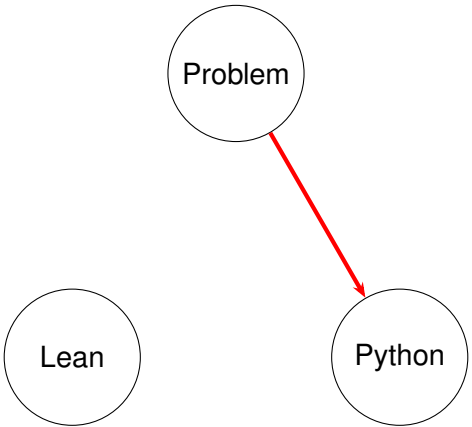
typical coding model



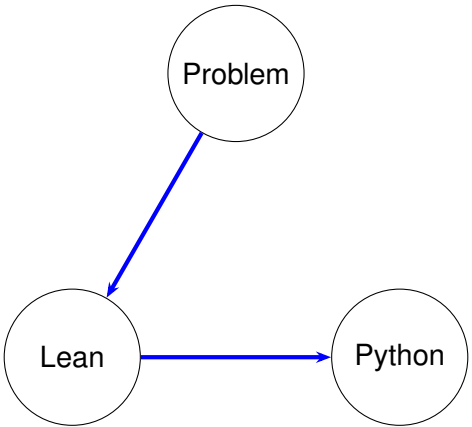
CLOVER: Ask an LLM interesting questions

- Do these unit tests match the doc string?
- Does this doc string summarize the code?
- . . .
- 6 questions in all
- Key idea in CLOVER by Sun, Sheng, Padon and Barrett.
  (They actually used formalization instead of unit tests)

Lean short cut



Lean short cut



I call this thinking in Lean

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But why should it help?

Sapir-Whorf Hypothesis

- My coauthors think in logic / math; I think in heuristics. We think differently.

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Sapir-Whorf Hypothesis

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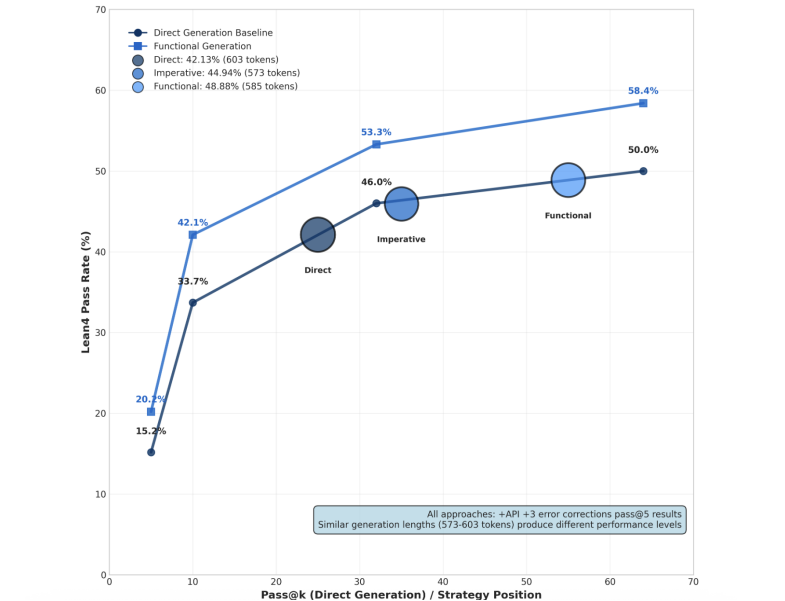
- Sapir-Whorf: the language you use shapes how you think.
  - Famous (and wrong) example: “Eskimos have 200 words for snow” — considered discredited.
- Sapir-Whorf-*Foster*: English vs. Chinese doesn’t matter — but English vs. *logic* is a big change. So is imperative vs. functional.

Great SF on the idea: Delany’s Babel-17*, Elgin’s* Native Tongue*, Kagan’s* Hellspark*.*

What if we do chain-of-though in Lean?


Performance vs Generation Length: Multi-Step Reasoning Shows Similar Lengths, Different Result
Direct Generation Baseline
Functional Generation
Direct: 42.13% (603 tokens)
Imperative: 44.94% (573 tokens)
Functional: 48.88% (585 tokens)
58.4%
53.3%
50.0%
46.0%
42.1%
33.7%
20.2%
15.2%
Direct
Imperative
Functional
All approaches: +API +3 error corrections pass@5 results
Similar generation lengths (573-603 tokens) produce different performance levels
Lean4 Pass Rate (%)
Pass@k (Direct Generation) / Strategy Position
0
10
20
30
40
50
60
70

Thinking functionally wins — for every model

- *BRIDGE*: 178 problems, 5 LLMs.
- Functional reasoning vs. direct Python generation:
  - $\sim$1.5$\times$ higher Lean correctness (pass@5) ✔
  - up to 2$\times$ more sample-efficient
- Holds across Claude, DeepSeek, Llama, Qwen.

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- Holds across Claude, DeepSeek, Llama, Qwen.
- And it's *learnable*: fine-tuning on functional reasoning traces beats code-only fine-tuning by $\sim$1.5$\times$.

Why functional helps

Functional prompting fixes exactly the errors Lean checks:

- Syntax errors: 45% $\rightarrow$ 12%
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- Fewer type errors

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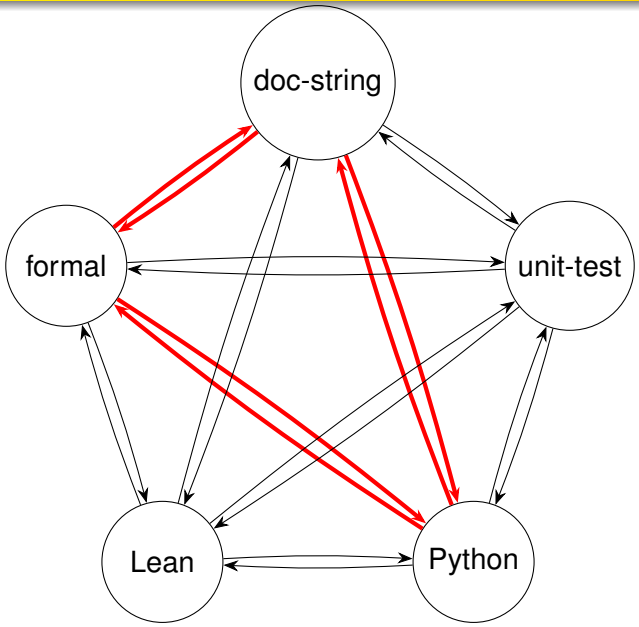
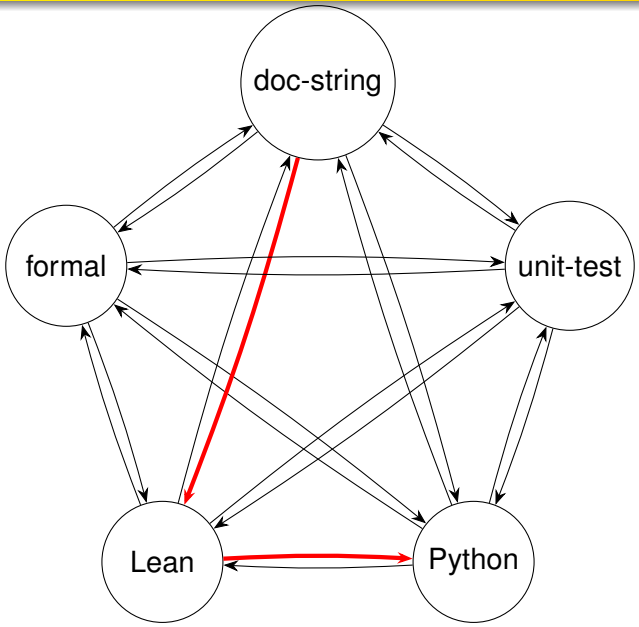
Not just more tokens: a length-matched “Double Lean” control (same budget) still loses, 15.2% vs. 20.2%. The win is the *representation*, not the verbosity.

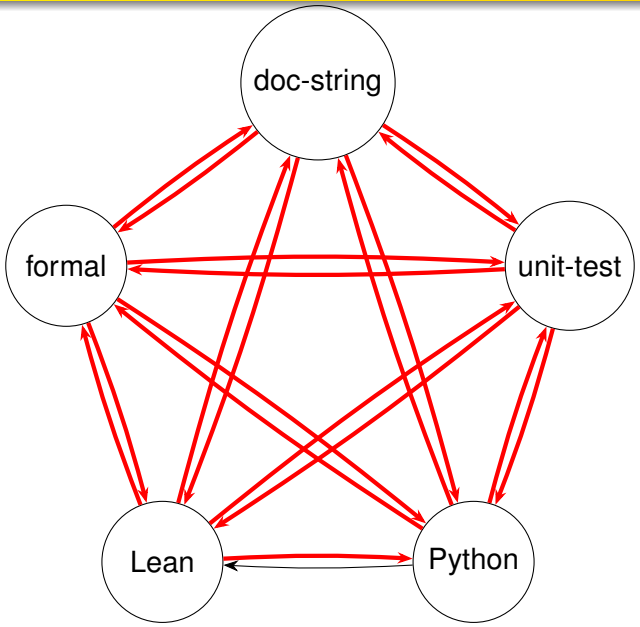
CLOVER: the starting point



Our work



The Dream: Putting them all together in one system



Manifest destiny: an agent that thinks in Lean

- Problem to solve? *Code.*
- What should it talk about? *Theorems.*
- Use Lean as the programming language:
  - so functional it won't call a function a "function" until you prove it isn't partial.

Manifest destiny: an agent that thinks in Lean

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- What should it talk about? *Theorems.*
- Use Lean as the programming language:
  - so functional it won't call a function a "function" until you prove it isn't partial.
- So I built one: **l3m** — a Claude-CLI-shaped agent, written in Lean, cloned in two weeks.
- Its trusted computing base is the Lean kernel, not a prompt.

l3m: safety you can *prove*

In **tame** mode, these are kernel-checked theorems, not promises:

- ✔ Never writes outside its own directory.
- ✔ Leaks at most 1000 bits to the web per session.
- ✔ Every action it takes can be undone.
- ✔ High-entropy strings (API_KEYs) are redacted on input.

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The trust report is a build artifact: every claim is a ProvenTheorem, a ManifestAxiom, or an explicit WorldClaim about the OS.
[manifests]

When `bash` is needed: back to IP

- Raw shell escapes the typed, provable surface — so the agent must *argue* for it.
- It runs an interactive proof (with a proxy verifier) for why `bash` is necessary.
  - Often it argues itself into a simpler, tame call instead.
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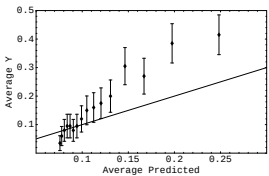
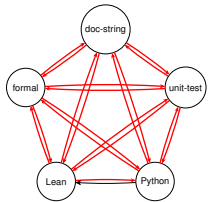
  - If it can't decide, it throws the question to me.
  - I rarely have to answer.
- Calibration let it *know what it doesn't know*; IP let it *prove what it does*; Lean let it *think in logic*.
- Each surface — `bash`, `git`, web, every project and scale — stays *separately* calibrated (multi-calibration).
- I now live in tame mode.

Close LLM Collaborators:

- Joao Sedoc [NYU]: Human evaluation and IP
- Carson Eisenach [Amazon]: RLMF, Lean
- Robert Joseph [Cal Tech], Udaya Guha [Amazon]: Lean
- Yuda Song [CMU]: mathematics
- Dominique Perrault-Joncas [Amazon]: Computer and human eval
- Omer Gottesman [Amazon], Lyle Ungar [U Penn], Emma Brunskill [Stanford], Amy Zhang [UT]: Education

THANKS!





(slides are at deanfoster.net)

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