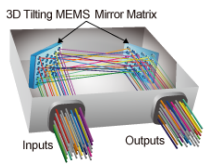
LLM code better when they think functionally

Dean Foster, Amazon

October 7, 2025



My background: Before LLMs

- Professor of Statistics
  - Regression: variable selection, “big data”
  - NLP: parsing, eigenwords before word2vec
  - Game theory: Calibration, fairness
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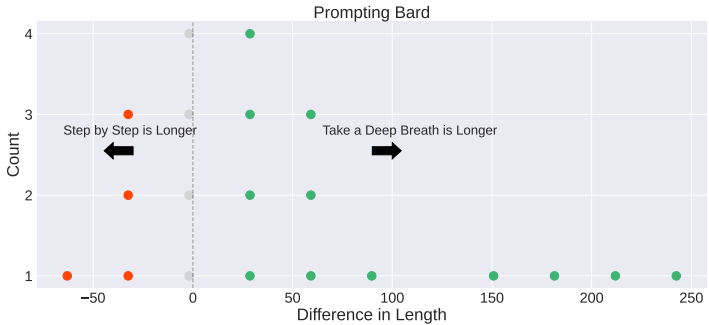
  - Created systems to control buying, cross docks and placement
  - While the supply chain caught up with RL, my team started working on LLMs as a side project

My background: After LLMs

- What I've been doing for the past 2 years:
  - Engineering: fighting communication bottlenecks (LLMs require bandwidth)
  - Alignment: defending LLMs using game theory ([1])
  - Optimization: picking the batch size ([2])
  - Applications: tutoring children ([3], [4])
  - Theory: Chain of thought lifts LLMs from TC^0 to PSPACE ([5])
  - RL: generate and test for self improvement ([6])
- What I'll talk about today:
  - Automatic reasoning, programming and Lean
  - Lean: getting LLMs to think differently

LLMs do better with more thinking



Contrasting native LLMs vs Chain of Thought

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(rephrased) An LLM can not answer questions in PSPACE.

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Other versions:

Theorem (Malach 2023)

A linear LLM can be trained to mimic a Turing machine using chain-of-thought.

Theorem (Giannou, Rajput, Sohn, Lee, Lee, and Papailiopoulos 2023)

Looped Transformers are general computers.

What should LLM’s think about?

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Lean!

What is *Lean*?

- Lean is used for formalizing mathematics:
  - Terry Tao is fascinated by Lean
  - He did a math project with 100s of people
  - They could all write Lean proofs
  - The proofs were “checked” by simply being type checked in Lean

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  - They could all write Lean proofs
  - The proofs were “checked” by simply being type checked in Lean
- The entire Cambridge undergraduate mathematics has been formalized in Lean
  - extensive library (mathlib) exists
  - Formalization of latex to lean can be done (But is hard)

What is *Lean*?

- Lean is a functional programming language:
  - Lean is a feature-complete functional programming language
  - It is a pure language: No side effects at all
  - This makes it easy to prove theorems about the code you write

What does is *Lean* math look like?

The square root of a prime is irrational with the start of its proof in lean:

```
example {m n p : ℕ} (nnz : n ≠ 0) (prime_p : p.Prime) : m ^ 2 ≠ p * n ^ 2 := by
  intro sqr_eq
  have nsqr_nez : n ^ 2 ≠ 0 := by simpa
  have eq1 : Nat.factorization (m ^ 2) p = 2 * m.factorization p := by
```

What is *Lean*?

- How is mathematics connected to programming?
  - “lambda calculus” was in mathematics before it was Lisp
  - Turing machines are mathematics and programs
  - Lean implements another form of logic based on type theory

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- How is mathematics connected to programming?
  - “lambda calculus” was in mathematics before it was Lisp
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  - Lean implements another form of logic based on type theory
- A theorem in Lean is a type
- A proof in Lean is an example of that type

What *Lean* isn't:

Lean is not a theorem prover

- Everyone knows 3SAT can represent any logic problem
- But no one writes 3SAT to describe their problems
- SMT solvers are generic tool
- Lean is not an SMT
  - It is a proof assistant
  - Humans write the proofs, and Lean checks it

What *Lean* isn't:

Lean is not axiomatic mathematics

- The axioms need to be added (called mathlib)
- So Lean is very small and has been proven to be a correct engine
- Lean doesn't make errors–axioms might!

Amazon's connection to *Lean*

- We have the Leo de Moura (the creator of Lean) on our Automatic reasoning team
- But we have 100 other scientist doing automatic reasoning (largest in the world)
  - They find bugs in hardware
  - They Find security leaks
  - They create provable correct translations crypto code
  - They create the trust that AWS users require

What does *Lean* code look like? (here is Quick Sort)

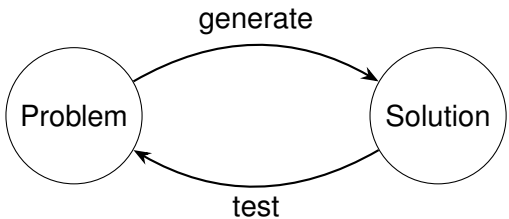
```
def qsort.F {α} (lt : α → α → bool) : Π (x : list α),
  (Π (y : list α), length y < length x → list α) → list α
| []     IH := []
| (h::t) IH := begin
    induction e : partition (λ x, lt h x = tt) t with large small,
    have : length small < length (h::t) ∧ length large < length (h::t),
    { rw partition_eq_filter_filter at e,
      injection e,
      subst large, subst small,
      constructor;
      exact nat.succ_le_succ (length_le_of_sublist (filter_sublist _)) },
    exact IH small this.left ++ h :: IH large this.right
  end
```

Writing lean code is hard;

checking it is easy

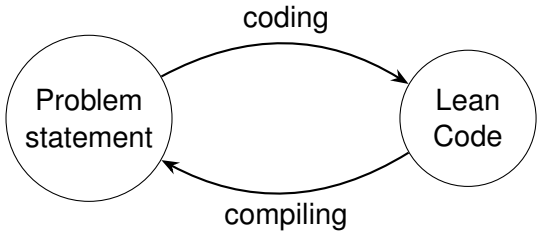
Generate and test

- Classic idea:
  - Generation is easier than testing
  - The gap can be HUGE:
    - Suppose NP $\approx$ EXP
    - Generation is takes exponential time
    - Testing takes polynomial times
- Nicely fits into mathematics
  - Problem to solution is generation
  - Solution to problem is testing

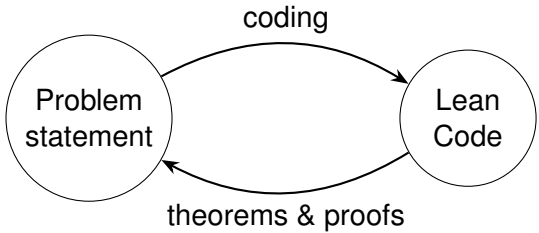

generate
Problem
Solution
test

Generate and test

- Fits nicely into LLMs
  - Have one LLM generate a solution
  - Have a different one test the solution
  - Improve the generation
- We found disappointing results ([6])
  - Saturates after a few rounds
  - Effectively doubling the data size
  - Far from a polynomial – exponential split
  - Better models have a bigger gap, so the future might still be rosy for this approach


coding
Problem statement
Lean Code
compiling

Only checks for termination.

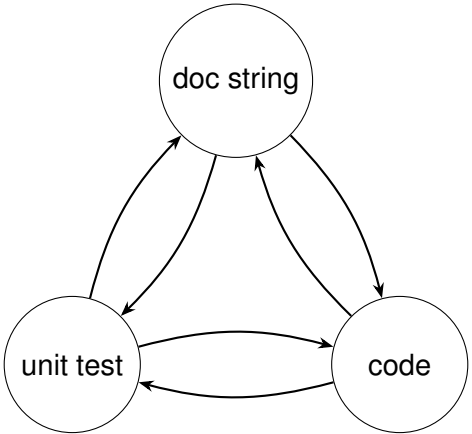

coding
Problem statement
Lean Code
theorems & proofs

Both directions require LLMs.

Give LLMs problems they can more easily solve

- Have LLMs:
  - Write the goal of the program (aka doc string)
  - Write unit tests
  - Write code
- Have them check consistency between them
- No compiling necessary! All done by LLMs

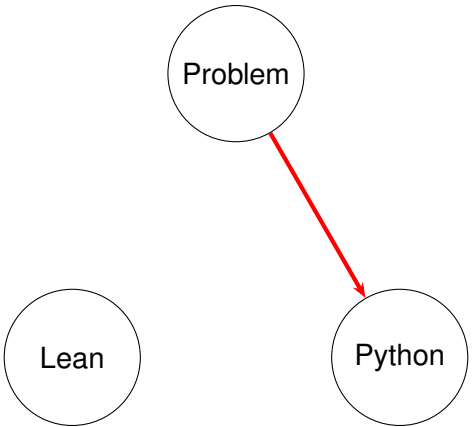
typical coding model



CLOVER: Ask an LLM interesting questions

- Do these unit tests match the doc string?
- Does this doc string summarize the code?
- . . .
- 6 questions in all
- Key idea in CLOVER by Sun Sheng Padon and Barrett.
  (They actually used formalization instead of unit tests)

Lean short cut



I call this thinking in Lean

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But why should it help?

Sapir-Whorf Hypothesis

- Some of my coauthors think in Mathematics

Sapir-Whorf Hypothesis

- Some of my coauthors think in Mathematics
- I don't!
  - I think in heuristics
  - I treat mathematics like an empirical science
- We think differently

Sapir-Whorf Hypothesis

- Sapir-Whorf hypothesis says that the language we use influences the way we think.
  - Example claim by Frank Boas: “Eskimos have 200 words for snow so they understand it better.”

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- Great SF though:
  - Early: Samuel R. Delany’s *Babel 17*
  - Hopeful: Suzette Haden Elgin’s *Native Tongue*
  - Less extreme: Janet Kagan’s *Hellspark*

Sapir-Whorf Hypothesis

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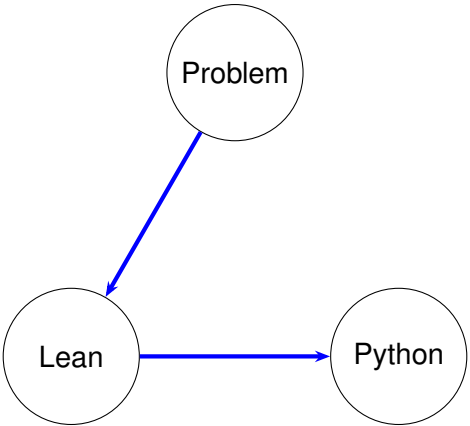
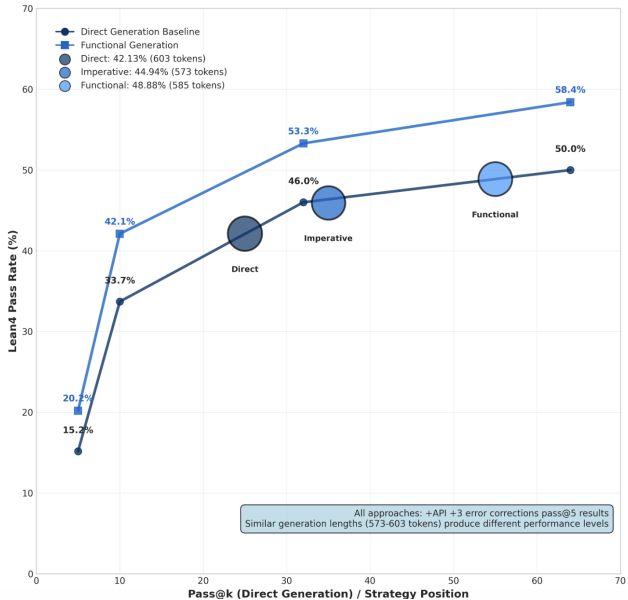
Sapir-Whorf Hypothesis

- Changing English to Chinese doesn't matter
- Changing English to Logic is a big change
- Changing imperative coding to functional coding is a big change

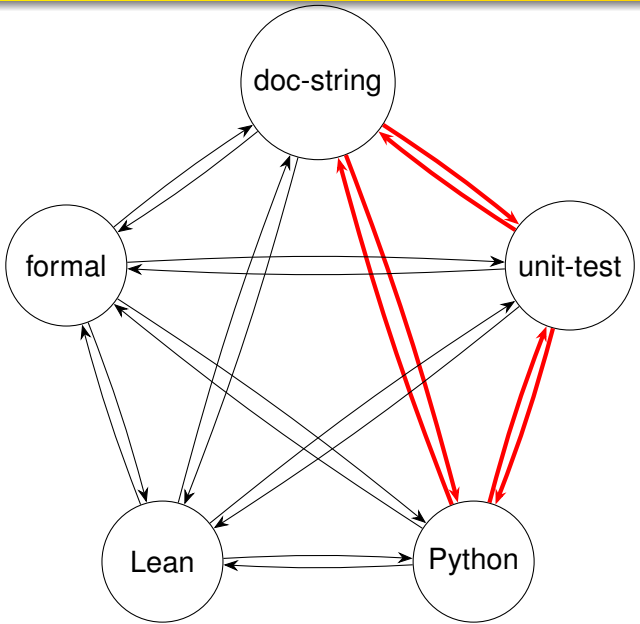
What if we do chain-of-though in Lean?

Lean short cut

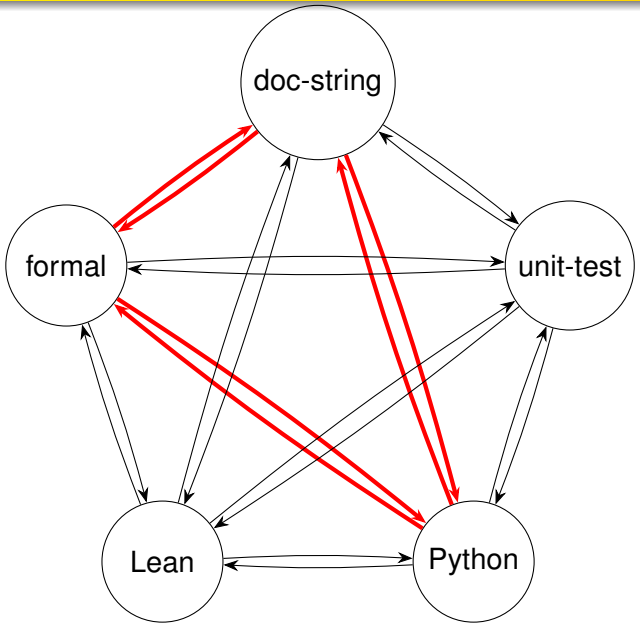



Performance vs Generation Length: Multi-Step Reasoning Shows Similar Lengths, Different Result
Direct Generation Baseline
Functional Generation
Direct: 42.13% (603 tokens)
Imperative: 44.94% (573 tokens)
Functional: 48.88% (585 tokens)
58.4%
53.3%
50.0%
46.0%
42.1%
33.7%
20.2%
15.2%
Direct
Imperative
Functional
All approaches: +API +3 error corrections pass@5 results
Similar generation lengths (573-603 tokens) produce different performance levels
Lean4 Pass Rate (%)
Pass@k (Direct Generation) / Strategy Position
0
10
20
30
40
50
60
70

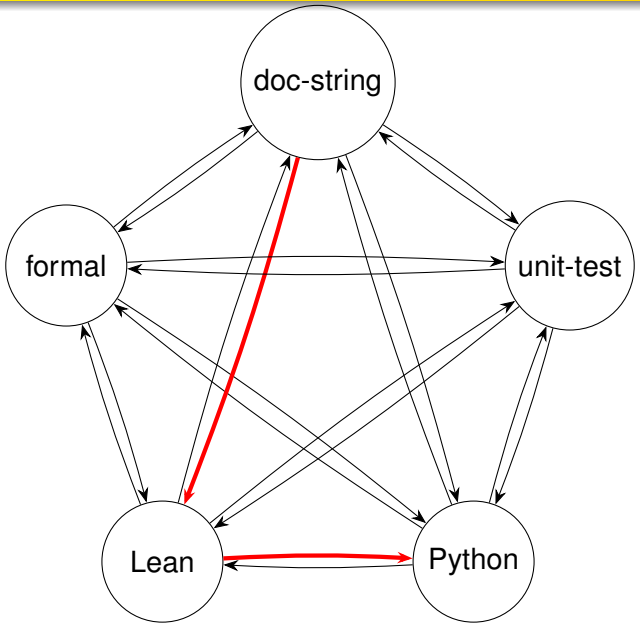
Starting point



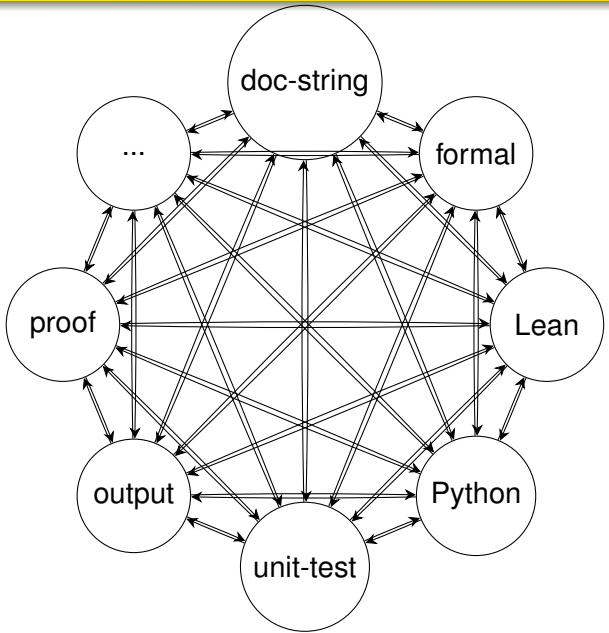
CLOVER



Our work



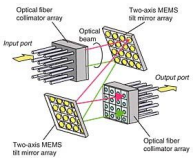
The dream



Close LLM Colaborators:

- Robert Joseph [intern and Cal Tech] Lean
- Carson Eisenach [NYC] RLMF, Lean
- Udaya Guha [NYC $\rightarrow$ AWS] Lean
- Dhruv Madeka [NYC $\rightarrow$ GDM] communication
- Omer Gottesman [NYC] education
- Riccardo Savorgnan [NYC $\rightarrow$ NYU] Lean
- Sham Kakade [Amazon Scholar and Harvard] Batch size and education
- Alex (Hyunji) Nam [intern and Stanford] Education
- Yuda Song [intern and CMU] mathematics
- Dominique Perrault-Joncas [Seattle] Computer and human eval
- Kari Torkkola [Seattle] Fine tuning
- Joao Sedoc [NYU] Human evaluation and theory (see him Friday!)
- Lyle Ungar [U Penn], Emma Brunskill [Stanford], Amy Zhang [UT]: Education
- . . .

THANKS!


Optical fiber collimator array
Two-axis MEMS tilt mirror array
Optical beam
Input port
Output port
Two-axis MEMS tilt mirror array
Optical fiber collimator array

Citations (slides at deanfoster.net and amazon.science)

"Principal / agent theory for LLMs and alignment," —. ([1])

"How Does Critical Batch Size Scale in Pre-training?" Zhang, Morwani, Vyas, Wu, Zou, Ghai, —, Kakade. ([2])

"What is the Value of Human-Level AI to Education?," Madeka, —, Kakade. ([3])

"Efficient RL for optimizing conversation level outcomes with an LLM-based tutor," Nam, Gottesman, Zhang, —, Brunskill, Ungar. ([4])

"A TCS look at LLMs," — at MTI-LLM. ([5])

"Mind the Gap: Examining the Self-Improvement Capabilities of Large Language Models Song, Zhang, Eisenach, Kakade, —, Ghai. ([6])

"Clover: Closed-Loop Verifiable Code Generation," Chuyue Sun, Ying Sheng, Oded Padon, Clark Barrett. (CLOVER)

"Progressive Formalization: A Multi-Representation Framework for Automated Verificatio Ceisen, George, —. Today's talk.