Bennett’s bound (1962) is usually stated for a bounded collection of \( n \) independent random variables \( U_1, \ldots, U_n \) with \( \sup |U_i| < M \), \( E U_i = 0 \), and \( \sum_i E U_i^2 = 1 \), and \( \tau > 0 \)

\[
P(\sum_i U_i \geq \tau) \leq \exp \left( \frac{\tau}{M} - \left( \frac{\tau}{M} + \frac{1}{M^2} \right) \log(1 + M\tau) \right).
\]

We will rewrite it and narrow its focus to \( n \) IID random variables \( X_1, \ldots, X_n \), which are bounded by 1, with \( \text{Var}(X_i) = \sigma^2 \). Then

\[
P(\bar{X} - E X \geq \gamma) \leq \exp \left( n\gamma - n(\gamma + \sigma^2) \log(1 + \gamma/\sigma^2) \right)
\]

Writing it differently:

\[
P(\bar{X} - E X \geq k\sigma^2) \leq \exp \left( n\sigma^2(\gamma - n(\gamma + \sigma^2) \log(1 + \gamma/\sigma^2)) \right)
\]

Or in its most traditional form, if \( x \ll \sigma \sqrt{n} \), expanding the log

\[
P \left( \frac{\bar{X} - E X}{\sigma/\sqrt{n}} \geq x \right) \leq \exp \left( -x^2/2 + x^3/(6\sigma\sqrt{n}) + O(x^4/n\sigma^2) \right)
\]

Or even more crudely \( x < .3\sigma \sqrt{n} \), then

\[
P \left( \frac{\bar{X} - E X}{\sigma/\sqrt{n}} \geq x \right) \leq \exp \left( -x^2/2(1 - x/\sigma \sqrt{n}) \right)
\]