Bennett's bound (1962) is usually stated for a bounded collection of n independent random variables $U_1, \ldots, U_n$ with $\sup |U_i| < M$, $E\,U_i = 0$, and $\sum_i E\,U_i^2 = 1$, and $\tau > 0$

$$P(\sum_i U_i \geq \tau) \leq \exp\left(\frac{\tau}{M} - \left(\frac{\tau}{M} + \frac{1}{M^2}\right)\log(1 + M\tau)\right).$$

We will rewrite it and narrow its focus to n IID random variables $X_1, \ldots, X_n$, which are bounded by 1, with $\mathrm{Var}(X_i) = \sigma^2$. Then

$$P(\overline{X} - EX \geq \gamma) \leq \exp\big(n\gamma - n(\gamma + \sigma^2)\log(1 + \gamma/\sigma^2)\big)$$

Writing it differently:

$$P(\overline{X} - EX \geq k\sigma^2) \leq \exp\big(n\sigma^2(k - (k+1)\log(k+1))\big)$$

Or in its most traditional form, if $x \ll \sigma\sqrt{n}$, expanding the log

$$P\left(\frac{\overline{X} - EX}{\sigma/\sqrt{n}} \geq x\right) \leq \exp\left(-x^2/2 + x^3/(6\sigma\sqrt{n}) + O(x^4/n\sigma^2))\right)$$

Or even more crudely $x < .3\sigma\sqrt{n}$, then

$$P\left(\frac{\overline{X} - EX}{\sigma/\sqrt{n}} \geq x\right) \leq \exp\left(-x^2/2(1 - x/\sigma\sqrt{n})\right)$$

- Bennett, G. (1962), "Probability inequalities for the sum of independent random variables," *JASA*, **57**, 33–45.