TCS for LLMs

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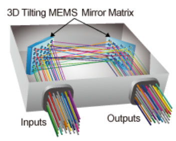
November 5, 2023



Real LLM discussion involve hardware

What makes modern LLMs work:

- GPUs
- cache efficient access
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- communication between devices and instances

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It's hardware, hardware, and more hardware

What can theory add?

Examples of cool theory:

1. "Auto-Regressive Next-Token Predictors are Universal Learners"
2. "SGD learning on neural networks: leap complexity and saddle-to-saddle dynamics"
3. saddle point escape
4. Many papers on two layer network theory
5. Many paper on the first step of SGD
6. μP
7. Matyroshka

And only 6 and 7 offer practical advice

Our goal: Useful theory

This talk: useful TCS for LLMs

I'll present 3 short ideas with implications for real NNs:

1. complexity of chain of thought
2. trap door functions
3. statistical degrees of freedom

Idea #1:

Chain of thought

Bad question:

Is $\sqrt{2\pi} \stackrel{?}{>} e$?

Good question:

Work out both sides of $\sqrt{2\pi} \stackrel{?}{>} e$, then say if it is true.

Best question:

Take a deep breath and work out both sides of $\sqrt{2\pi} \stackrel{?}{>} e$, then say if it is true.

Theorem (Merrill and Sabharwal 2023)

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Theorem (F. and Madeka 2023, Folk theorem 2024)

Using chain of thought reasoning, an LLM can solve any problem in PSPACE.

Implication #1:

Feed the out of one NN into another NN during training

Tiered model

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- Middle tiers:
  - training: Using history and rollout, predict next word
  - generates new roll outs
- Top tier:
  - Reads all roll outs and history
  - training: predictions the next word
  - inference: uses predictions to generate actual word

Idea #2:

One way functions

One way functions

A one way function is one where $f(x)$ is easy to compute, but $f^{-1}(y)$ is hard to compute.
Examples:

- Cryptography
- Effectively random functions
- P vs NP

Causal mask

We process words sequentially in a transformer LLM.

- Not as extreme as say in a LSTM
- Still, all values are "time stamped"
  - Every node in a transformer has a time stamp
  - It only depends on tokens that came before that time stamp
- Say more...

Extremely small embedding

Theorem

Suppose each layer i has nodes t such that $N_{i,t} = f(N_{i-1,t}, N_{i-1,t-1})$. Suppose further that $N_{i,t} \in R^1$. Then there exists polynomials with low complexity that take exponential computation under this restriction.

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Example of what is going on:

- $b_t = b_{t+1}^2$
- Easy to compute from right to left
- takes one multiply at each step
- computing left to right requires raising to power x^{2^T}

Harder example:

- $b_t = \alpha_t + \beta_t b_{t+1} + \gamma_t b_{t+1}^2$
- Easy to compute from right to left
- computing left to right is a very complex polynomial

Extremely small embedding

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Key point:

- It needs to have a small context window
- Any fixed size will have hard examples
- We can compute R2L easy, but L2R is hard

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How to attack the theorem:

- copy all data to the time t
- do all the computation
- Now as easy as R2L, but requires a huge embedding dimension

Implication #2:

"encoder" plus transformer network

Insert description here

Idea #3:

Statistical batch vs computational batch

Statistics independence

Palm masked out the first 10% of their tokens in every batch.

- Worked with a batch of 2000 tokens
- $Y_1, \ldots, Y_{t-1}$ used to predict Y_t
- But only for $t = 201, 202, \ldots, 2000$
- First 200 tokens not predicted in this batch

Statistics independence

Our encoder / decoder trick:

- Encodes 9000 tokens
- predicts next 1000 tokens
- First 9000 not predicted in this batch

Implication #3:

Stride length $\neq$ window length

Statistics independence

Trick to use more data:

- L = batch size
- S = "stride" (the number of predictions made)
- Use batchs 0, L

Summary

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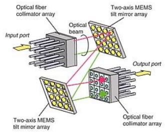
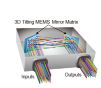
This argued for the following modifications to LLM foundation models:

- rollout aware network
- An encoder-decoder model {**Dean:** *What word do we use to replace encoder?*}
- better sampling of tokens

THANKS!



Pointers (we'll drop this .pdf in the chat)



MEMS

- Big bench: 100s of hard problems.
- PaLM and PaLM2 solve BigBench and professional exams
- Magical hour talk (Sebastien Bubeck)
- Magical 15 minute talk (Kahn of Kahn academy)
- Prompt Engineering (Andrew Ng's Prompt engineering)
- nanoGPT on github (build an LLM from scratch in 2 hours)

LLM requirements:
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(Need about 1000 to 10,000 A100s or H100s for 3 or 4 months.
So maybe 3 million dollars up to 100 million dollars.)

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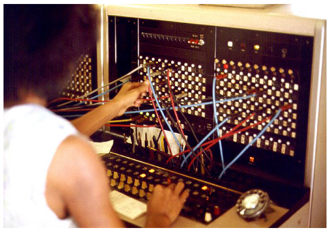

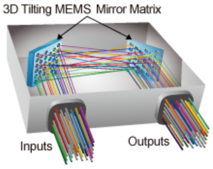

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nanoGPT github/video

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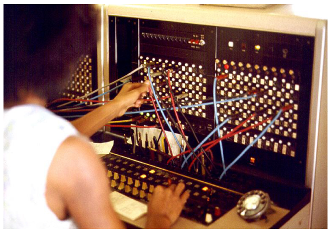
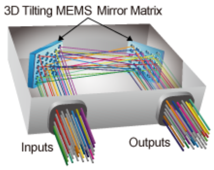
LLM find patterns

$$
\begin{aligned}
\overline{L}(\text{random guessing}) &= 15 = \log_2(60,000) \\
\overline{L}(\text{unigrams word frequency}) &= 11.7 = \log_2(3300) \\
\overline{L}(\text{bigrams (aka Markov)}) &= 8.8 = \log_2(500) \\
\overline{L}(\text{gzip (LZ compression)}) &= 8.2 = \log_2(300) \\
\overline{L}(\text{small LLM}) &= 7.5 = \log_2(200) \\
\overline{L}(\text{Humans}) &\approx 4 \\
\overline{L}(\text{LLM}) &= 3.6 = \log_2(12)
\end{aligned}
$$

(All in bits per token. I did the small LLM. Shannon, Cover/King did the human subjects estimation.)

Point to point is faster than packets





MEMS